Mongolia's TOP-20 Index Risk Analysis, Pt. 3

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In the third part of our risk report on TOP-20 Index, Mongolia's main stock market indicator, we focus on modelling the right tail of the distribution of returns using the two main approaches in extreme value theory: the extrema, or block maxima (BM), and the peek-over-threshold (POT). This way, we achieve a near-perfect fit of the whole distribution of returns.

Overview

In this report we provide a correct specification of the right tail of the distribution of TOP-20 Index returns, the place where extreme gains are located.

Previously surveyed models [4], although excellent at describing the left tail of the distribution of returns, offered a generally poor fit to the right tail of the same. An accurate prediction of extreme losses is, of course, very important for investors, but in the presence of positive skewness and negative leverage, so is that of extreme gains.

For our purpose we resort, once again, to extreme value theory. In particular, we look at two complementary methodologies: the extrema (or block maxima, BM) approach, and the peek-over-threshold (POT) approach [1]. The focus of both is on the tail index parameter ξ, which defines the shape of the tail distribution. The extrema approach looks for the optimal ξ able to give a precise representation of the distribution followed by the maxima in a sample of data, one among the so-called generalised extreme value (GEV) distributions: Weibull, Gumbel, and Fréchet.

The peek-over-threshold approach looks for the best ξ to characterise the distribution followed by exceedances over a large threshold, the generalised Pareto (GP) distribution. Although based on different sets of assumptions, the two methods give similar results, especially when applied to shocks (demeaned returns, divided by a time-varying standard deviation), instead of raw returns. We will provide evidence of this fact in a Monte Carlo experiment.

The joint use of an efficient conditional volatility model, such as GJR-VT, a good density, such as the asymmetric t, up to a high quantile, and extreme value theory for the right tail, leads to a near-perfect fit of the whole distribution of returns.

Data

We use TOP-20 Index daily GJR-VT shocks calibrated from daily log returns* (August 13, 2007 – March 3, 2017). The latter are price returns (they only consider capital appreciation and omit dividends), gross of fees, expenses, and taxes. We use shocks, instead of returns, because the former are more stable and less subject to variability, as they effectively neutralise autocorrelation and volatility clustering which tend to inflate returns in periods of directional mFarkets and which may inject bias into the data fitting process.

Shortcomings of the previous model

Before discussing extreme value theory, we briefly review the GJR-VT asymmetric t model introduced in the second part of this risk report, pointing out its weakness in fitting the right tail of the distribution of TOP-20 returns. Let us separately analyse its components.

GJR-VT, the conditional variance model, is fully specified by ω, the intercept, α and β, respectively, the ARCH and GARCH coefficients, and θ, the leverage parameter. The asymmetric t distribution is completely determined by d_1, the number of degrees of freedom, and d_2, the skewness parameter.

Maximum-likelihood (MLE) calibration leads to optimal values ω = 1.07E–05, α = 0.1954, β = 0.8023, θ = –0.1193, d_1 = 4.06, and d_2 = 0.08.

The model provides a near-perfect fit up to a high quantile, above which it visibly deviates from empirical data (Figure 1). To determine the quantile, we may compute the absolute distance between the sorted GJR-VT shocks and the corresponding values predicted by the asymmetric t distribution, and ask such distance to be smaller than a certain threshold, say 0.30. The t quantile above which this condition is violated is the one we are looking for.

For immediate detection of violations we may plot the logarithm of the absolute distance (the transformation is chosen to avoid spikes in the graph). When the absolute distance between two data points is very small, its logarithm is negative and large because the right-sided limit of the logarithm to zero is minus infinity. As a consequence, we expect good models to produce graphs with very few data points higher than ln(0.30) ≈ –1.2040 (Table 1). The asymmetric t misaligns eleven, and severely misaligns (distance > 0.50) six

Figure 1: QQ-plot of GJR-VT shocks against the asymmetric t distribution

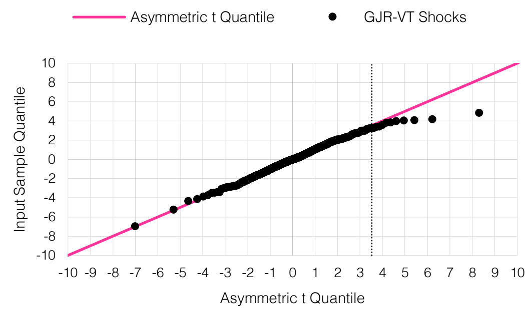
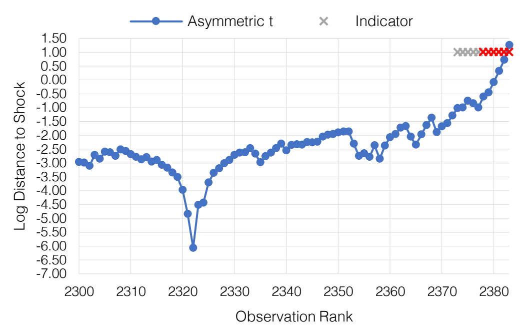


Table 1: Asymmetric t distribution right tail fit	
Absolute distance allowed	0.30
Logarithmic distance	–1.2040
No. violations (severe)	11 (6)
Threshold quantile (obs.)	3.5272 (2373rd)

Figure 2: Logarithmic absolute distance between GJR-VT shocks and t quantiles



shocks (Figure 2; gray and red crosses detect points in which the distance is more than allowed). The threshold quantile is 3.5272, and corresponds to the 2373rd observation; all shocks above this mark belong to the right tail, which we are now going to model with EVT.

Generalised extreme value and the block maxima approach

The Fisher-Tippett theorem[a] states that, if the maxima in a sample are i.i.d. (independent and identically distributed) and if there exist a location parameter μ (mean), real number, a scale parameter $\psi > 0$ (standard deviation), and a cumulative distribution function H such that the limit distribution of standardised maxima converges to H then, depending on the optimal value of the tail parameter ξ, H must be one of the three standard extreme value distributions:

$\xi < 0$ (thin tails): Weibull distribution

$\xi = 0$ (exponential): Gumbel distribution

$\xi > 0$ (fat tails): Fréchet distribution

When this occurs, the cumulative distribution function of the maxima is said to *belong to the domain of attraction of H*.

For instance, the cdf of the Gaussian density belongs to the domain of attraction of the Gumbel distribution, that of the Student's t to the Fréchet distribution.

To verify which distribution the maxima belong to, we first divide the whole sample of GJR-VT shocks into 40 subgroups, each one covering 60 trading days (three months of daily data) and, for each subgroup, we find the maximum value: what we get is a sample of 40 block maxima which should be approximately i.i.d., as required (Table 2, Figure 3).

Then, before calculating the optimal value of ξ, we plot the quantiles of the sorted, standardised maxima against the Gumbel quantiles. If the distribution of maxima belongs to the domain of attraction of the Gumbel, the QQ-plot should be roughly linear; if it belongs to that of either the Weibull or the Fréchet, the plot should be convex or concave, respectively. So:

linear QQ-plot: Gumbel distribution

convex QQ-plot: Weibull distribution

concave QQ-plot: Fréchet distribution

The empirical distribution of GJR-VT shock maxima seems to belong to the domain of attraction of the Weibull, as the QQ-plot is convex, apart from the last segment (Figure 4). Thus, we expect the tail parameter $\xi > 0$.

We estimate the optimal values for μ, ψ, and ξ through MLE. $\mu = 2.4067$, $\psi = 0.7608$, and $\xi = -0.1785$, as expected, so that the shape parameter $\alpha = 1/\xi = -5.6010$ (Table 3). Also, the distribution has an upper bound, corresponding to the quantile 6.6677: this means no predicted shock can be higher than that.

Table 2: Distribution of shock maxima statistics	
No. of groups	40
Data in each group	60 shocks (3m)
Min./max.	1.1259; 4.8129
Mean	2.7339
Standard deviation	0.8321
Skewness	0.3316
Excess kurtosis	–0.2725

Figure 3: Time distribution of shock maxima

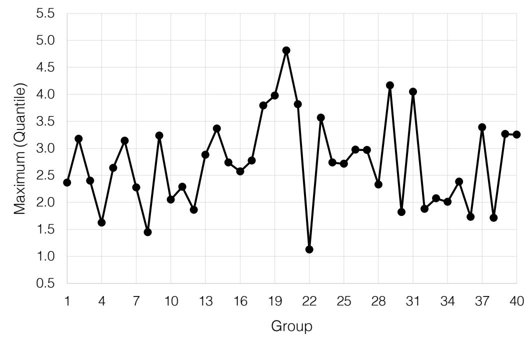
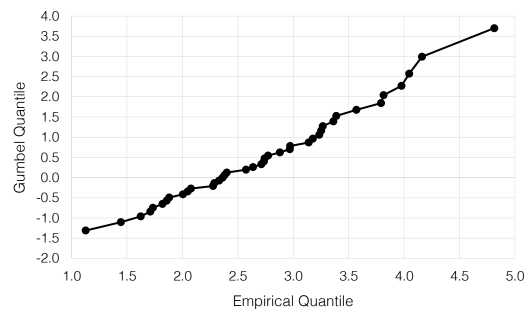


Figure 4: Empirical quantile of the shock maxima against the Gumbel quantile



We provide the optimal Weibull density juxtaposed with the histogram of maxima (Figure 5). The peak of the density occurs at μ, and it is where the data locate on average, or cluster; the width of the same is given by ψ; the thickness of the right tail by ξ; overall shape by α.

With the optimal parameters at hand, we proceed to apply the Weibull fit to the right tail of the distribution of sorted GJR-VT shocks. We may freely select the quantile from which to start, the only requirement is that it should be high enough (say, greater than 2.50). We choose the one above which the distance between the shocks and the asymmetric t quantile is larger, in absolute value, than that between the shocks and the GEV quantile. To ease the comparison, we plot the logarithmic distances, and put a cross on data for which this condition is met (Figure 6). Apart from two isolated observations (nn. 2320-1), for which the improvement is in any case negligible, the Weibull distribution does not offer a good fit until data point 2350, above which it gives a much higher conformity to empirical values than the asymmetric t. The threshold quantile corresponding to such data point is 2.5290[b]. Joint use of the asymmetric t distribution up to, and the GEV above, the threshold quantile, leads to a near-perfect fit of the whole set of GJR-VT shocks (Figure 7).

The main drawback of the maxima approach is that to preserve the validity of the i.i.d. assumption it only considers the largest values in each subsample, ignoring data which may still provide useful information on the shape of the right tail. The peek-over-threshold approach, instead, focuses on the exceedances over a large threshold and, thanks to the smaller loss of information, it may yield a better estimate of the right tail.

Table 3: GEV parameter estimation, MLE	
ξ (csi)	–0.1785
μ (mu)	2.4067
ψ (psi)	0.7608
α (alpha) = $1/\xi$	–5.6010
Maximised log-likelihood	–48.26
Which distribution?	Weibull
Upper bound	6.6677

Figure 5: Distribution of GJR-VT shock maxima against the Weibull distribution

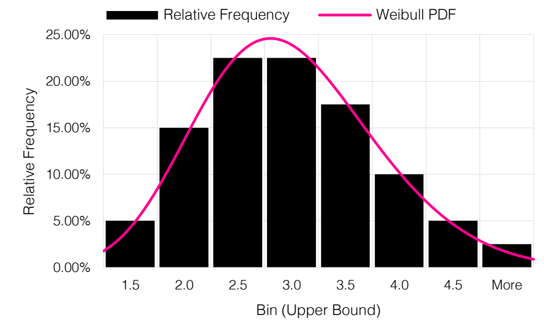


Figure 6: Logarithmic distance of the theoretical quantiles to empirical data

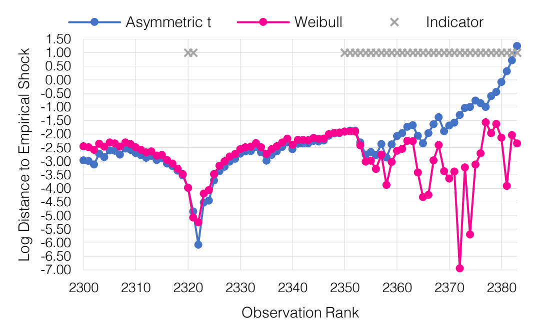
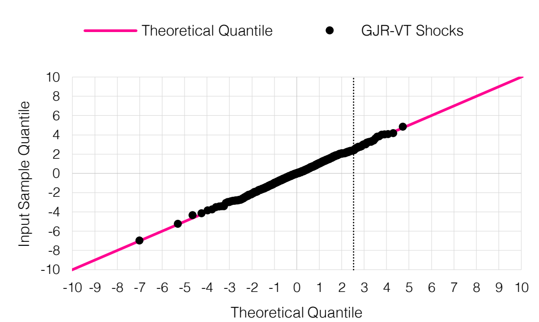


Figure 7: QQ-plot of GJR-VT shocks against the asymmetric t with GEV right tail



Generalised Pareto distribution and the peek-over-threshold approach

If the cumulative distribution function of GJR-VT shocks is in the domain of attraction of the extreme value distribution H, then the excess distribution function F_u (the cdf of all the realisations above a user-defined threshold u) can be approximated, for u large, by the generalised Pareto distribution[c].

The GPD is completely specified by two parameters, whose meaning is the same as in the maxima approach: ξ, the tail index, and ψ, a positive scaling function of the threshold u (there is no parameter μ). An increase of ξ, with ψ unvaried, makes the right tail thicker and the shoulder steeper. An increase of ψ, with ξ unvaried, makes the distribution more spread out. The optimal values for ξ and ψ crucially depend on that of the threshold u, which should both be high enough to ensure that the limit distribution of the excess distribution function is actually a GPD, and low enough so that there is a sufficient amount of data for a stable estimation of the parameters. We choose u = 2.50, so as to have 31 exceedances (~1.30% of total observations) for model calibration (Table 4). We retrieve optimal values for ξ and ψ through MLE.

$\xi = -0.2978$, $\psi = 0.9093$, so that $\alpha = -3.3582$.

We provide the optimal tail density juxtaposed with the histogram of exceedances (Figure 8). The threshold quantile seems to be appropriately selected because the distances between the sorted shocks and the theoretical values from the GPD are much smaller (Figure 9). The combined use of the asymmetric t distribution up to, and the GPD above, such quantile, leads to a near-perfect fit of the entire set of GJR-VT shocks (Figure 10). Also, the proxy given by the GPD is almost identical to that provided by the GEV.

Table 4: GPD parameter estimation, MLE

Threshold quantile	2.50
No. exceedances	31
Fraction of the sample	1.30%
ξ (csi)	–0.2978
ψ (psi)	0.9093
α (alpha) = $1/\xi$	–3.3562
Maximised log-likelihood	–18.82

Figure 8: Distribution of GJR-VT shock exceedances against the GPD distribution

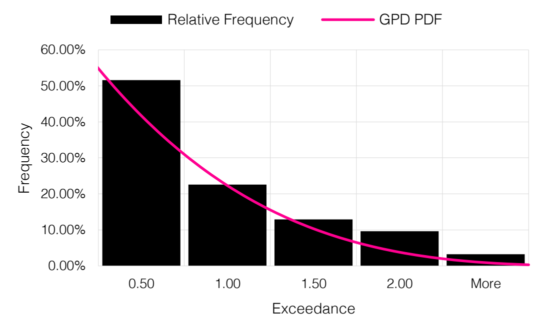


Figure 9: Logarithmic distance of the theoretical quantiles to empirical data

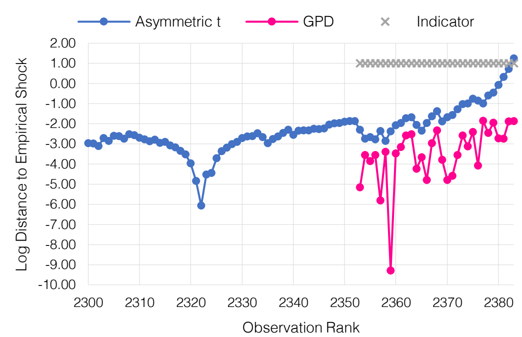
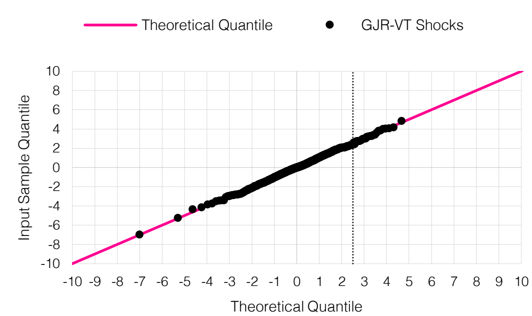


Figure 10: QQ-plot of GJR-VT shocks against the asymmetric t with GPD right tail



Although the peek-over-threshold approach does not sacrifice as many data points as the extrema approach does, it still suffers from two major drawbacks. The first one is the assumption of i.i.d.-ness of all returns (not only of the maxima over subsamples), which is hardly met in practice. The second one is the selection of the optimal threshold u, which inevitably falls on the user.

Model performance

To show that the two EVT models give very close results (the similarity was already apparent from the QQ-plots), we forecast the distribution of TOP-20 Index returns a month from now, using Monte Carlo and the inverse transform method [2] [3].

To simulate a GJR-VT asymmetric t model with GEV right tail:

1. We create 21 sets of 100,000 uniform random variables. We interpret these variables as probabilities, since they take value in [0,1];
2. We compute the asymmetric t quantiles corresponding to such probabilities, with different formulae depending on whether the latter fall below or above $(1 - d_2)/2$, with $d_2 = 0.08$ so that the threshold value is 0.4608;
3. If the asymmetric t quantile is greater than or equal to 2.5290, the value above which the GEV fit is superior, we switch to the Weibull quantile;
4. We calculate the vector of returns one day ahead by multiplying the first set of shocks with the most recent GJR-VT figure;
5. We build the vector of GJR-VT conditional variances using the squared simulated returns and the predicted level of volatility (multiplier: 0.6);
6. We repeat each step 21 times, then aggregate the figures to obtain a set

Table 5: MC-GEV investment statistics	
No. of simulations	100,000
Average return	–0.1749%
Volatility	5.8560%
Skewness	0.4558
Excess kurtosis	3.4651
Neg./pos. return frequency	53.51%; 46.49%
Average if neg./pos.	–4.24%; 4.51%
Freq. returns beyond ±10%	3.47%; 4.34%
Average if beyond ±10%	–13.27%; 14.48%

Figure 11: MC-GEV distribution of TOP-20 Index monthly returns

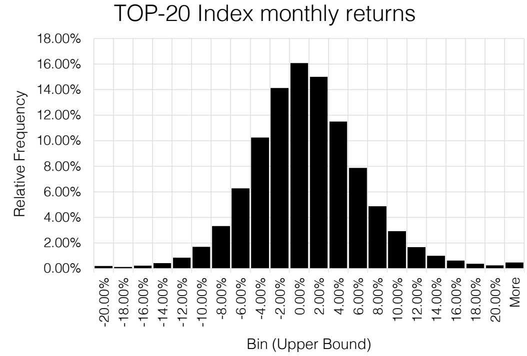
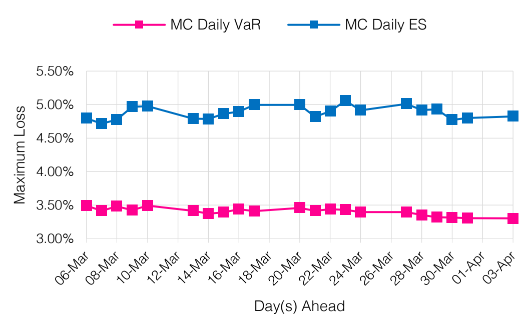


Table 6: MC-GEV monthly VaR and ES	
Value at Risk, 99%	13.77%
Expected Shortfall, 99%	17.72%
Portfolio value	MNT 1,000,000
MNT Value at Risk	MNT 128,632.11
MNT Expected Shortfall	MNT 162,364.65

Figure 12: MC-GEV term structure of Value at Risk and Expected Shortfall



of monthly TOP-20 Index returns;

7. We compute monthly VaR and ES, together with their term structures.

To simulate a GJR-VT asymmetric t model with GPD right tail, we proceed in the same way except for the third step, which is modified as follows:

3. If the asymmetric t quantile is greater than or equal to u = 2.50, the threshold above which the GPD is calibrated, we switch to the GPD quantile.

The two models produce very similar results. Both predict a negative average return, quite in line with the empirical evidence of the most recent six-year period [5] (Tables 5 and 7). Volatility is close to current market figures (~5-6%), positive skewness, positive excess kurtosis, and negative leverage are all present. The latter is visible in the higher frequency of losses (54-46), the larger average gain (0.3% more, in absolute value, than the average loss), the higher frequency of extreme gains (~1% higher than that of extreme losses), and the greater average extreme gain (1.2% more than the average extreme loss) (Figures 11 and 13). One-month ahead Value at Risk is around 13.7% (Tables 6 and 8), or between 3.30% and 3.50% per day (Figures 12 and 14). MNT VaR is around 128,500 per million invested (~EUR 49.20). Monthly Expected Shortfall is around 17.8%, or between 4.70% and 5.00% per day. MNT ES is around 162,500 per million (~EUR 62.20).

Comparison to the previous model

We can contrast the output from the hybrid models with that of the pure asymmetric t, as shown in Tables 25-26 in part two of the risk report. It is clear that the hybrid distributions offer a more precise estimate of the four moments: the predicted average return is negative and closer to the empirical figures of the past six years; volatility is dampened, thanks

Table 7: MC-GPD investment statistics	
No. of simulations	100,000
Average return	–0.1828%
Volatility	5.9007%
Skewness	0.5646
Excess kurtosis	6.1368
Neg./pos. return frequency	53.77%; 46.23%
Average if neg./pos.	–4.24%; 4.53%
Freq. returns beyond ±10%	3.47%; 4.36%
Average if beyond ±10%	–13.27%; 14.56%

Figure 13: MC-GPD distribution of TOP-20 Index monthly returns

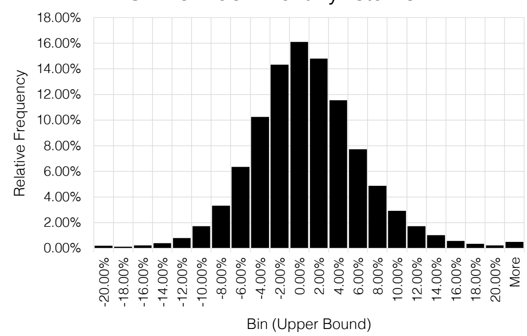
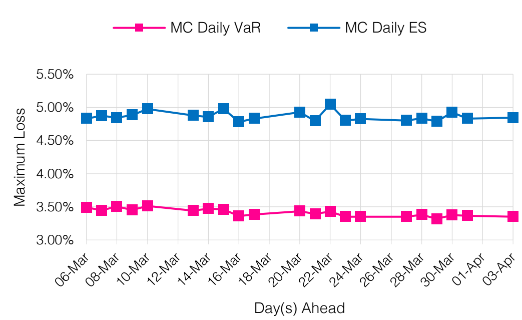


Table 8: MC-GPD monthly VaR and ES	
Value at Risk, 99%	13.63%
Expected Shortfall, 99%	17.77%
Portfolio value	MNT 1,000,000
MNT Value at Risk	MNT 128,436.02
MNT Expected Shortfall	MNT 162,786.23

Figure 14: MC-GPD term structure of Value at Risk and Expected Shortfall



to the lower dispersion of gains; skewness and excess kurtosis, still positive, are now smaller for the same reason. The relative frequency of losses and gains is unvaried because EVT simply improves the fit of the right tail, with no effect on the balance between positive and negative returns. What changes, however, are the average gain, the frequency of extreme gains, and the average extreme gain. All are now reduced, due to the lower dispersion of returns in the tail. The average return is 0.30% smaller, the frequency of extreme returns is 0.70% lower, and the average extreme return is 1.20% less.
Overall, we believe gains, especially large ones, to be much better specified than they were before.

Conclusion

We modelled the right tail of the distribution of TOP-20 Index returns using the main techniques in extreme value theory: the extrema, or block maxima, approach and the peek-over-threshold approach. Both look for the optimal value of ξ, the parameter governing the shape of the right tail. We found the Weibull distribution with $\xi = -0.1748$ and the GPD with $\xi = -0.2978$ to offer the nicest fit, which is also better than that provided by the asymmetric t, above the threshold quantile ~2.50. With the EVT improvement, we finally achieve a near-perfect fit of the whole density of TOP-20 Index returns.

The report is made for Standard Investment LLC by Federico M. Massari, a long distant volunteer risk analyst, using the sources provided.

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* All data from mse.mn. We modified the value of the close price recorded on August 13, 2010 from MNT 11,145.50 to MNT 10,145.50; the previous datum was most likely an outlier resulting from transcription error.

Bibliography

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[3] Glasserman, P.: *Monte Carlo Methods In Financial Engineering*, 2003, Springer Applications of Mathematics.

[4] Massari, F.M.: *Mongolia's TOP-20 Index Risk Analysis, Pts.1-2,* 2017, Standard Investment LLC.
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Available at: standardinvestment.mn/en

Notes

[a] Also known as the *Fisher-Tippett-Gnedenko theorem*, or as the first theorem in extreme value theory.

[b] Note that this is the quantile for the single shock, not for the maximum. It is obtained from the relationship $(1 - p^*) = (1 - p)^N$, with p^* being the probability that the maximum over a subsample is above a large quantile, $p = 1 - (2350 - 0.5)/\text{sample size} \approx 1 - 98.59\%$ being the probability that the single shock is above a large quantile, and $N = 60$ being the number of shocks in a subsample. The corresponding quantile for the maximum is 4.6752. See [1], paragraph 7.1.4, "Estimation of high quantiles".

[c] Also known as the *Pickands-Balkema-de Haan theorem*, or as the second theorem in extreme value theory.

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